Phase diagram of Ising model with Dzyaloshinskii-Moriya Interaction

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$$H = h_0(\vec{r_1}) + h_0(\vec{r_2}) + \frac{e^2}{|\vec{r_1} - \vec{r_2}|}$$

$$\langle \phi_a | \phi_b \rangle = 0$$

$$H_{exch} = -\sum_{ij} \left(J_{ijx} S_i^x S_j^x + J_{ijy} S_i^y S_j^y + J_{ijz} S_i^z S_j^z \right)$$

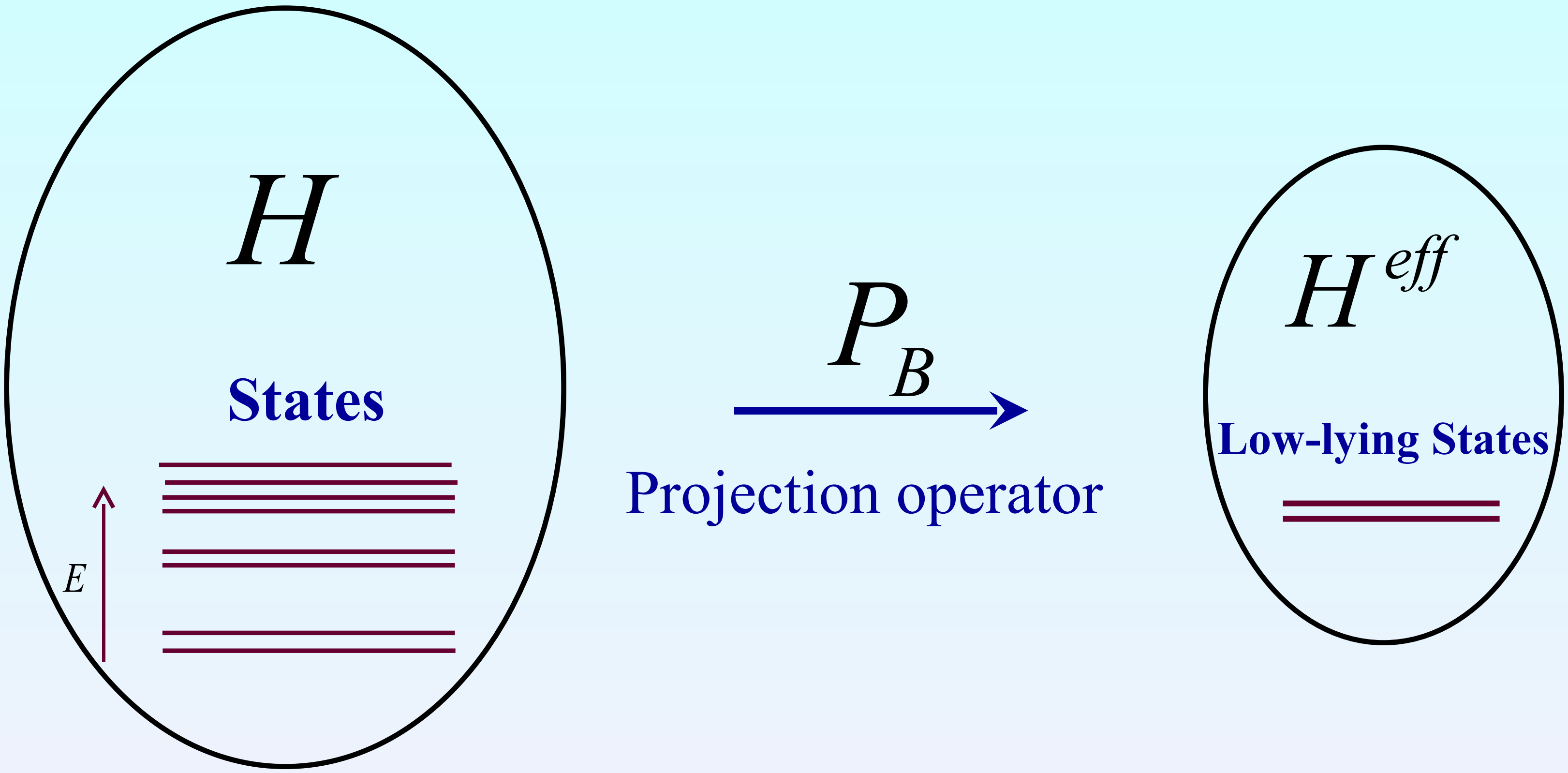
$$\langle \phi_a | \phi_b \rangle \neq 0$$

$$H_{exch} = \sum_{ij} \left(J_{ijx} S_i^x S_j^x + J_{ijy} S_i^y S_j^y + J_{ijz} S_i^z S_j^z \right)$$

$$J_{ij\alpha} \rangle 0 \quad (\alpha = x, y, z)$$

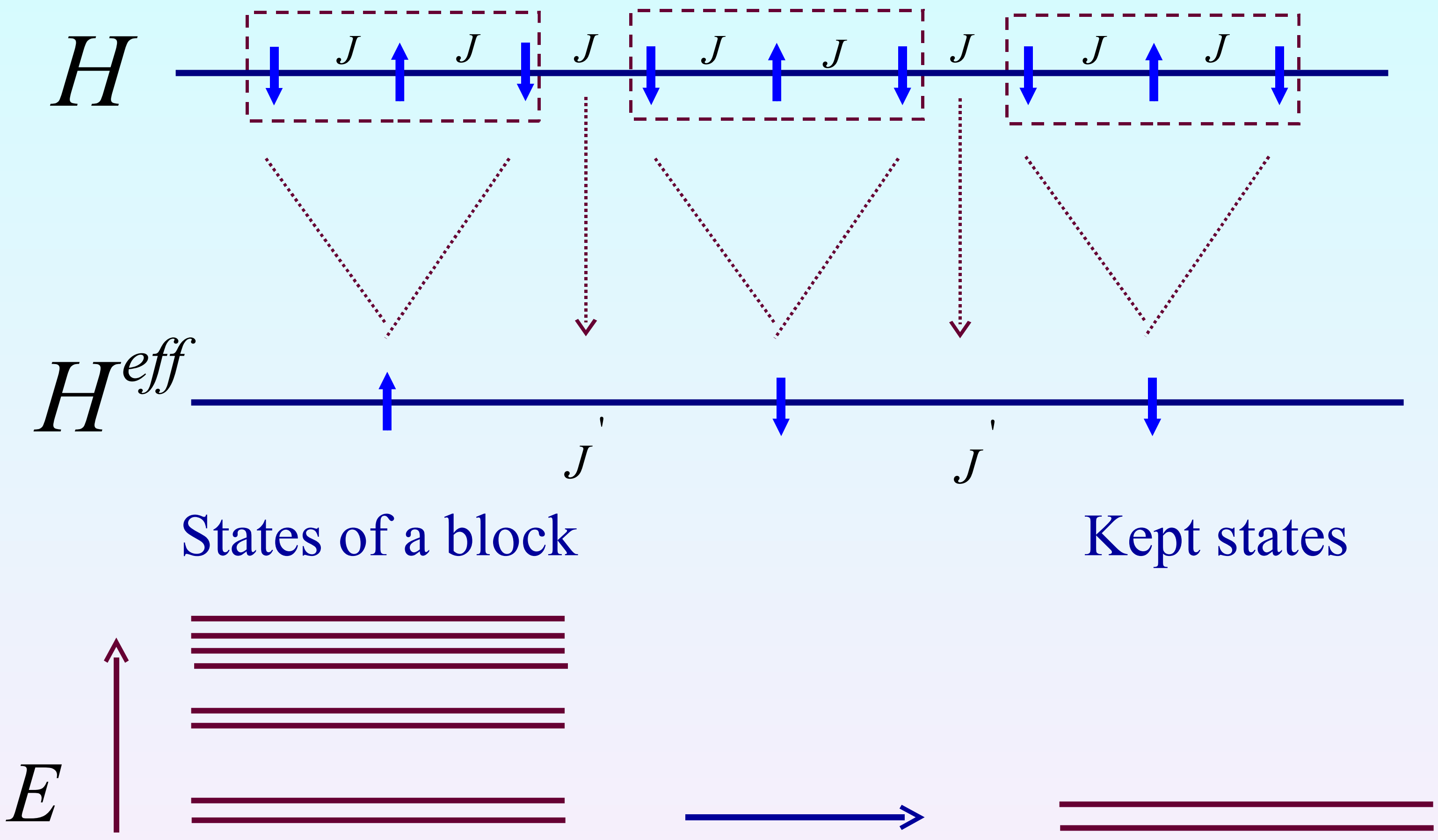
Renormalization group

The main idea of the RG method is the mode elimination or thinning of the degrees of freedom followed by an iteration which reduces the number of variables step by step until a more manageable situation is reached. We have implemented the Kadanoff's block method to do this purpose. Kadanoff's method, the lattice is divided into blocks which the Hamiltonian is exactly diagonalized. By selecting a number of low-lying eigenstates of the blocks the full Hamiltonian is projected onto these eigenstates which gives the effective (renormalized) Hamiltonian.



H
States
E
P_B
Projection operator
H^{eff}
Low-lying States

Quantum RG:



$$H = H^{B} + \lambda H^{BB}$$

$$H^{B} = \sum_{I=1} h_I^B$$

$$P_B = P_0 + \lambda P_1 + \lambda^2 P_2 + \ldots$$

$$H^{eff} = H_0^{eff} + \lambda H_1^{eff} + \lambda^2 H_2^{eff} + \ldots$$

$$H^{eff} = P_B \, H \, P_B$$

$$P_0^I = \sum_m^k |\psi_m\rangle\langle\psi_m|$$

$$P_0 = \bigotimes_{I=1}^{\frac{N}{n_B}} P_0^I$$

$$H_0^{eff} = P_0 H^B P_0 \qquad H_1^{eff} = P_0 H^{BB} P_0$$

RG of Ising Model with DM interaction:

$$H = \frac{J}{4}\left[\sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z + \vec{D} \cdot (\vec{\sigma}_i \times \vec{\sigma}_{i+1})\right], \quad \vec{D} = D\hat{k}$$

$$H = \frac{J}{4}\left[\sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z + D(\sigma_i^x \sigma_{i+1}^y - \sigma_i^y \sigma_{i+1}^x)\right]$$

$$h_I^B = \frac{J}{4}\left[\sigma_{1,I}^z \sigma_{2,I}^z + \sigma_{2,I}^z \sigma_{3,I}^z + D(\sigma_{1,I}^x \sigma_{2,I}^y - \sigma_{1,I}^y \sigma_{2,I}^x + \sigma_{2,I}^x \sigma_{3,I}^y - \sigma_{2,I}^y \sigma_{3,I}^x)\right]$$

$$H^{BB} = \frac{J}{4}\sum_{I=1}^{N/3}\left[\sigma_{3,I}^z \sigma_{1,I+1}^z + + D(\sigma_{3,I}^x \sigma_{1,I+1}^y - \sigma_{3,I}^y \sigma_{1,I+1}^x)\right]$$

$$\left|\psi_0\right\rangle = \frac{1}{\sqrt{2q(1+q)}}\left[2D\left|\downarrow\uparrow\uparrow\right\rangle + i(1+q)\left|\uparrow\downarrow\uparrow\right\rangle - 2D\left|\uparrow\uparrow\downarrow\right\rangle\right]$$

$$\left|\psi_0'\right\rangle = \frac{1}{\sqrt{2q(1+q)}}\left[2D\left|\downarrow\downarrow\uparrow\right\rangle + i(1+q)\left|\downarrow\uparrow\downarrow\right\rangle - 2D\left|\uparrow\downarrow\downarrow\right\rangle\right]$$

$$e_0 = -\frac{J}{4}(1+q)$$

$$\left|\psi_1\right\rangle = \frac{1}{\sqrt{2q(q-1)}}\left[2D\left|\downarrow\uparrow\uparrow\right\rangle - i(q-1)\left|\uparrow\downarrow\uparrow\right\rangle - 2D\left|\uparrow\uparrow\downarrow\right\rangle\right]$$

$$\left|\psi_1'\right\rangle = \frac{1}{\sqrt{2q(q-1)}}\left[2D\left|\downarrow\downarrow\uparrow\right\rangle - i(q-1)\left|\downarrow\uparrow\downarrow\right\rangle - 2D\left|\uparrow\downarrow\downarrow\right\rangle\right]$$

$$e_1 = -\frac{J}{4}(1-q)$$

$$\left|\psi_2\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\uparrow\right\rangle\right), \left|\psi_2'\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\downarrow\downarrow\uparrow\right\rangle - \left|\uparrow\downarrow\downarrow\right\rangle\right)$$

$$e_0 = 0$$

$$\left|\psi_3\right\rangle = \left|\uparrow\uparrow\uparrow\right\rangle \quad , \quad \left|\psi_3'\right\rangle = \left|\downarrow\downarrow\downarrow\right\rangle$$

$$e_3 = \frac{J}{2}$$

$$P_0^I = \left|\psi_0\right\rangle_{I\,I}\left\langle\psi_0\right| + \left|\psi_0^{'}\right\rangle_{I\,I}\left\langle\psi_0^{'}\right|$$

$$P_0^I \sigma_{1,I}^x P_0^I = -\frac{2D}{q}\sigma_I^y \,, P_0^I \sigma_{2,I}^x P_0^I = \frac{4D^2}{q(q+1)}\sigma_I^x \,, P_0^I \sigma_{3,I}^x P_0^I = \frac{2D}{q}\sigma_I^y$$

$$P_0^I \sigma_{1,I}^y P_0^I = \frac{2D}{q}\sigma_I^x \,, P_0^I \sigma_{2,I}^y P_0^I = \frac{4D^2}{q(q+1)}\sigma_I^y \,, P_0^I \sigma_{3,I}^y P_0^I = -\frac{2D}{q}\sigma_I^x$$

$$P_0^I \sigma_{1,I}^z P_0^I = \frac{1+q}{2q}\sigma_I^z \,, P_0^I \sigma_{2,I}^z P_0^I = -\frac{1}{q}\sigma_I^z \,, P_0^I \sigma_{3,I}^z P_0^I = \frac{1+q}{2q}\sigma_I^z$$

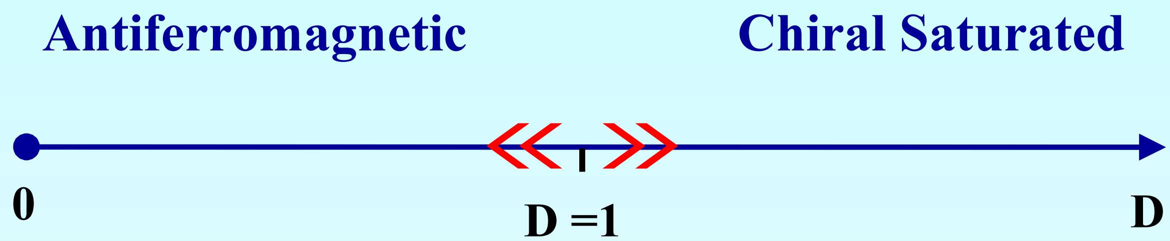
$$H^{eff} = \frac{J}{4}\left[\sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z - D(\sigma_i^x \sigma_{i+1}^y - \sigma_i^y \sigma_{i+1}^x)\right]$$

$$\sigma_i^z \to -\sigma_i^z \,, \sigma_i^y \to -\sigma_i^y$$

RG Equation:

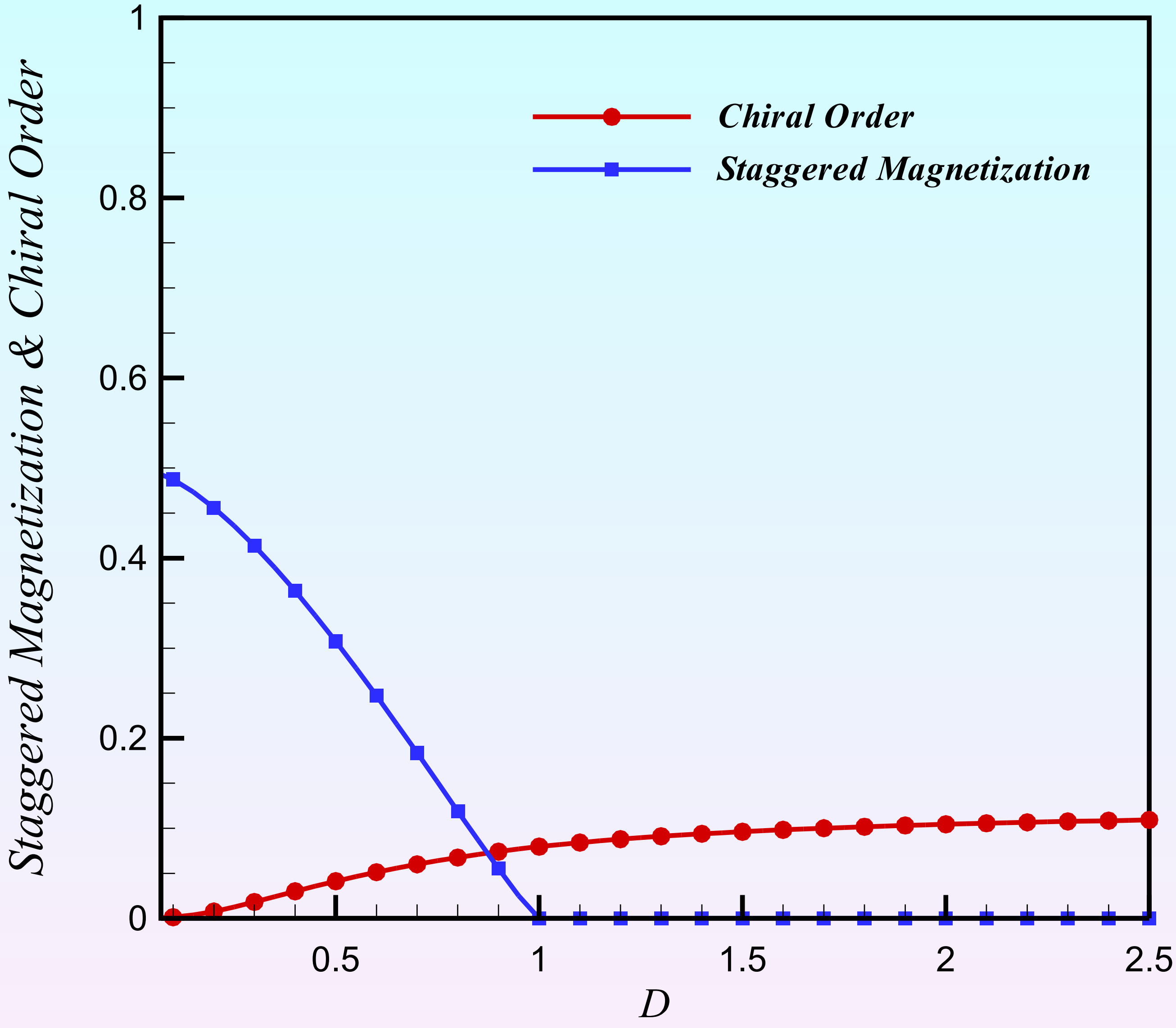
$$H^{eff} = \frac{J^{'}}{4}\left[\sum_{i=1}^{N}\sigma_i^z\sigma_{i+1}^z + D^{'}(\sigma_i^x\sigma_{i+1}^y - \sigma_i^y\sigma_{i+1}^x)\right]$$

$$J^{'} = J\,\frac{(1+q)^2}{2q}\,,\; D^{'} = \frac{16D^3}{(1+q)^2}\,,\; q = \sqrt{1+8D^2}$$



$$S_M = \frac{1}{N}\sum_{i=1}^{N}\langle 0|(-1)^i \sigma_i^{\alpha}|0\rangle$$

$$C_h = \frac{1}{N}\sum_{i=1}^{N}(\sigma_i^x \sigma_{i+1}^y - \sigma_i^y \sigma_{i+1}^x)$$



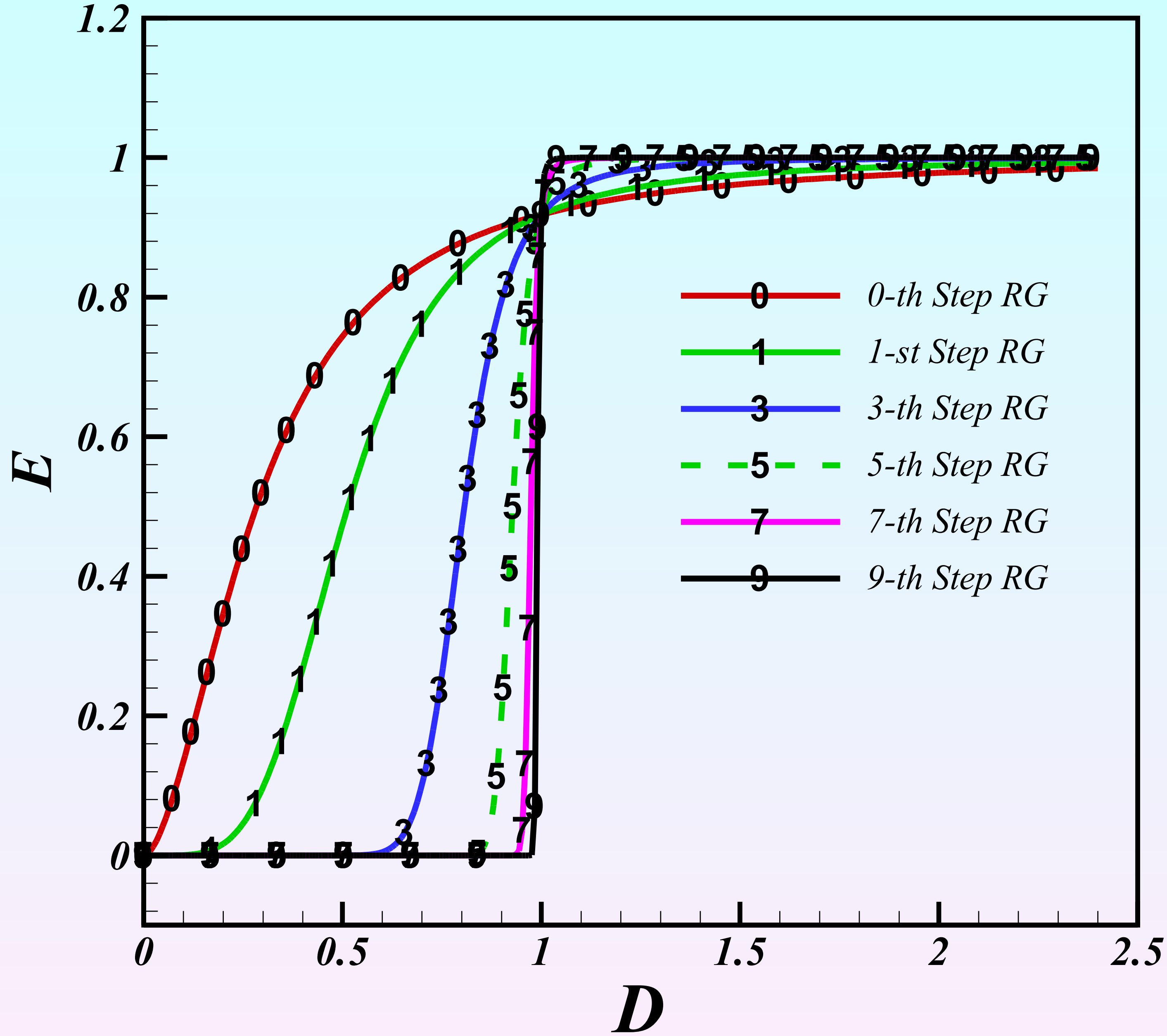
Chiral Order
Staggered Magnetization
Staggered Magnetization & Chiral Order
D
1
0.8
0.6
0.4
0.2
0
0.5
1
1.5
2
2.5

$$\rho = |\psi_0\rangle\langle\psi_0| \qquad \rho_2 = Tr_{13}\,\rho$$

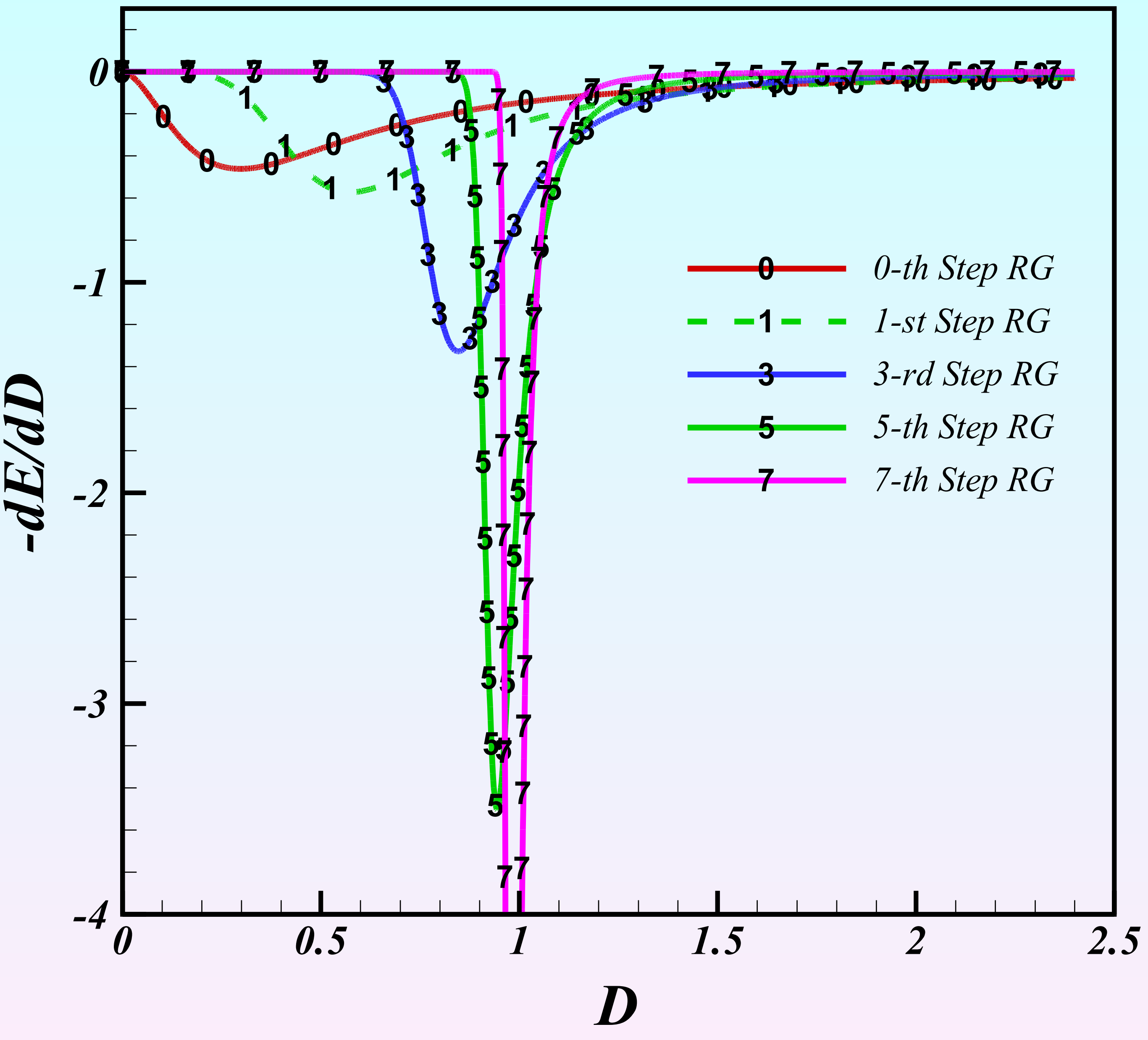
$$\rho_2 = \frac{1}{2q(1+q)}\begin{pmatrix} 8D^2 & 0 \\ 0 & (1+q)^2 \end{pmatrix}$$

$$E = -Tr\,\rho_2\,Log_2\,\rho_2$$

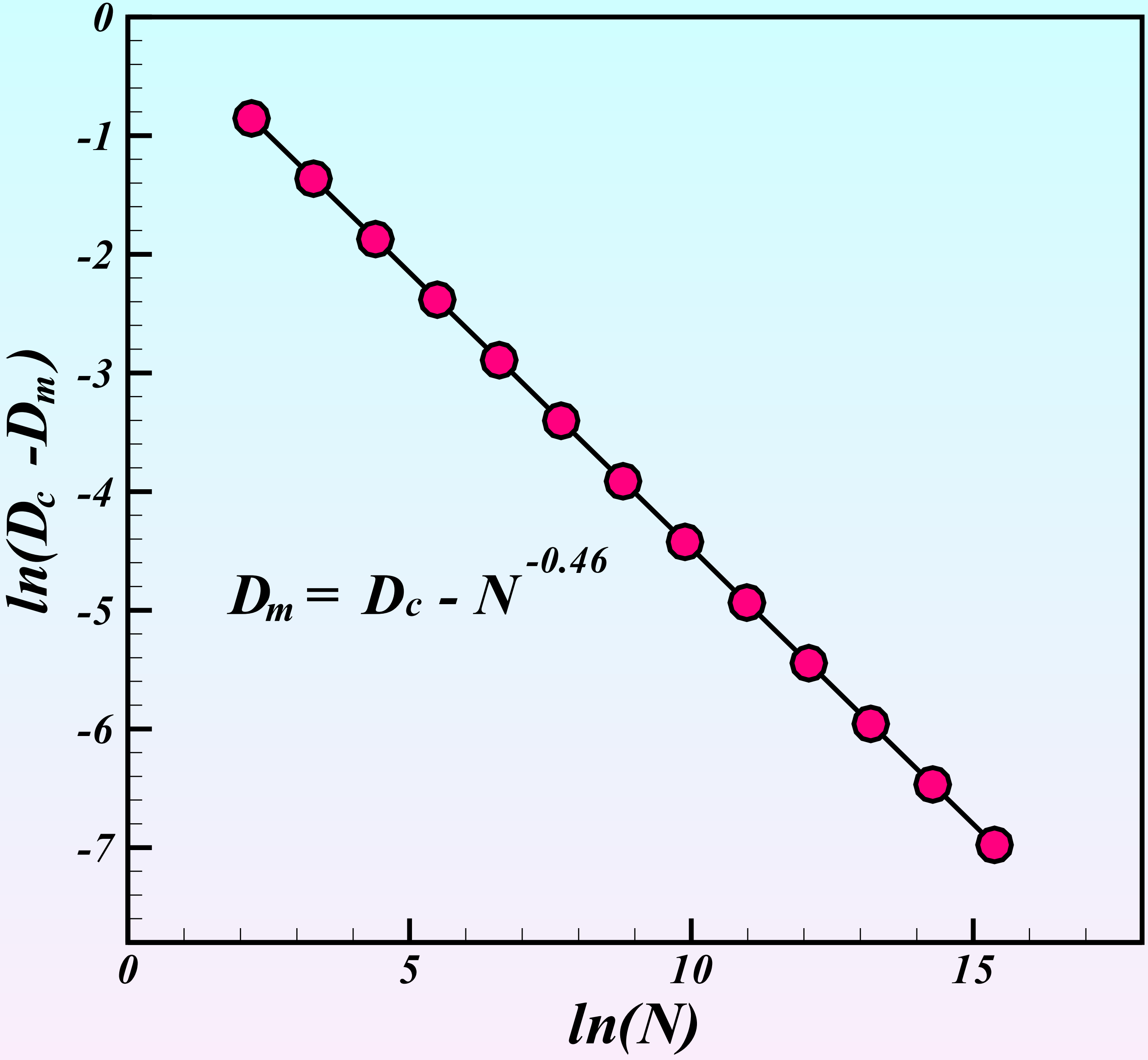
$$E = -\left(\frac{8D^2}{2q(1+q)}\right)\log_2\left(\frac{8D^2}{2q(1+q)}\right) - \left(\frac{(1+q)^2}{2q(1+q)}\right)\log_2\left(\frac{(1+q)^2}{2q(1+q)}\right)$$



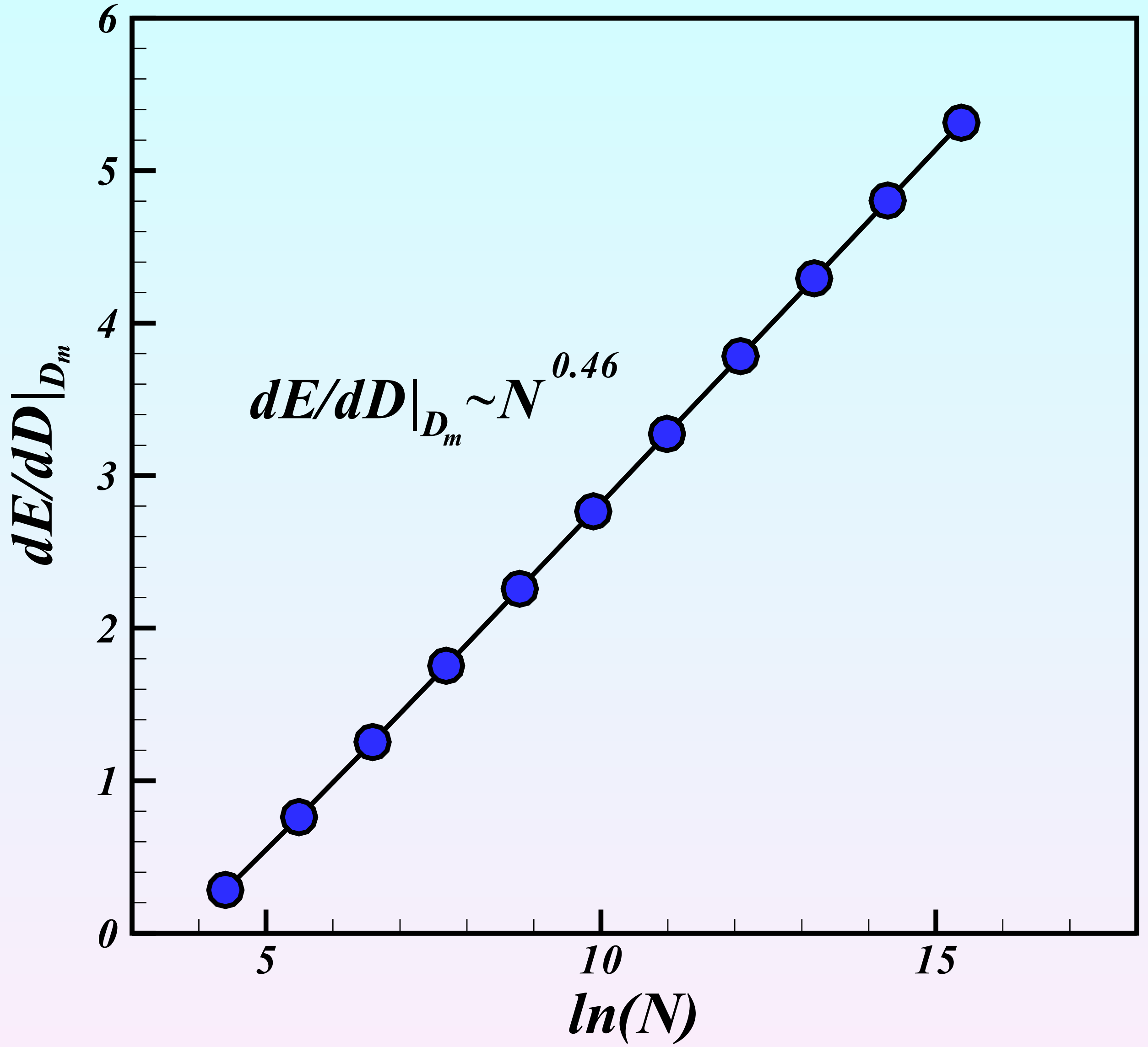
1.2
1
0.8
0.6
0.4
0.2
0
E
0
0.5
1
1.5
2
2.5
D
0-th Step RG
1-st Step RG
3-th Step RG
5-th Step RG
7-th Step RG
9-th Step RG



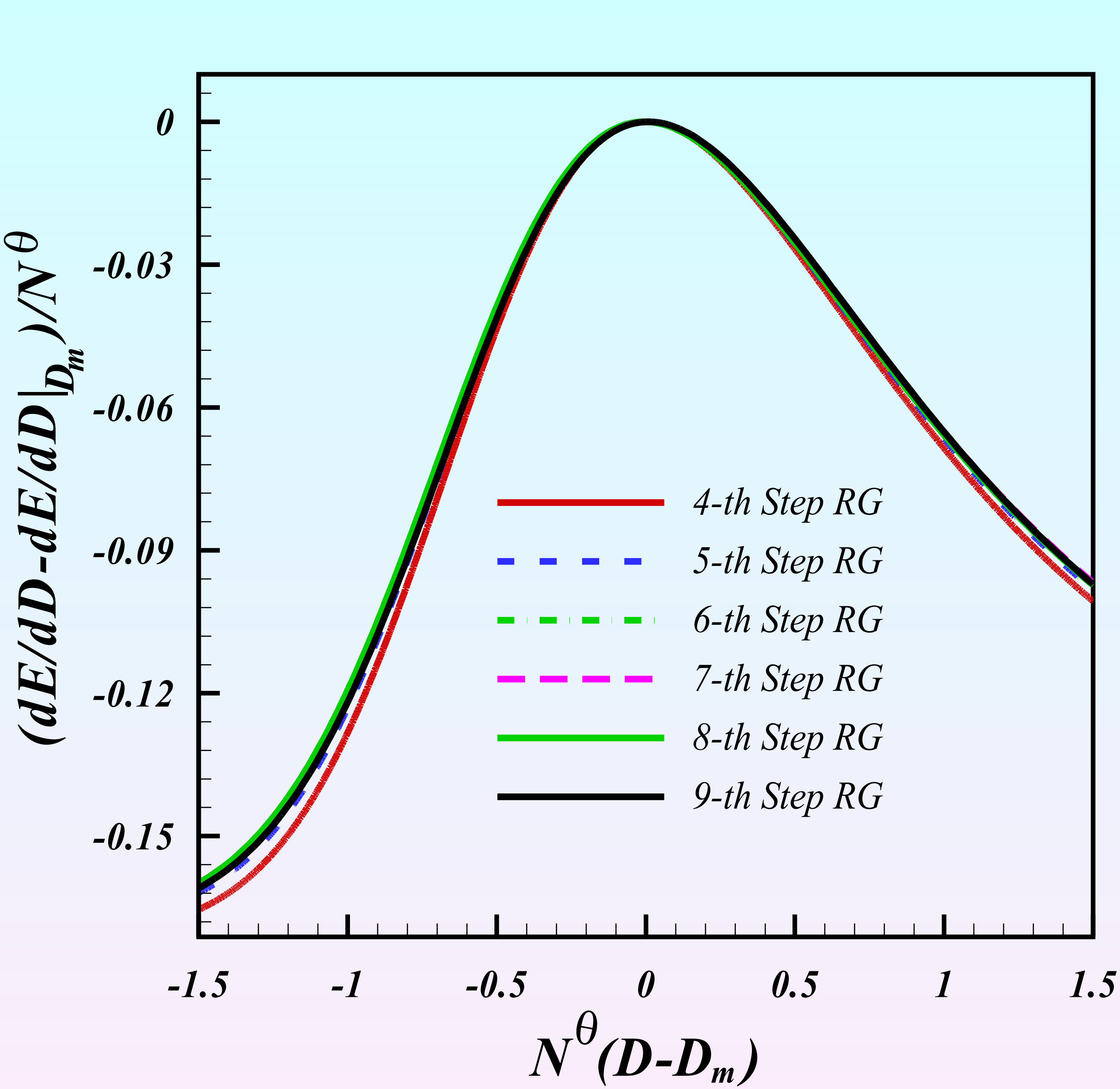
-dE/dD
D
0
-1
-2
-3
-4
0
0.5
1
1.5
2
2.5
0-th Step RG
1-st Step RG
3-rd Step RG
5-th Step RG
7-th Step RG



0
-1
-2
-3
-4
-5
-6
-7
0
5
10
15
ln(N)
ln(Dc -Dm)
Dm = Dc - N-0.46



$dE/dD|_{D_m} \sim N^{0.46}$
$dE/dD|_{D_m}$
$ln(N)$
0
1
2
3
4
5
6
5
10
15



0
-0.03
-0.06
-0.09
-0.12
-0.15
-1.5
-1
-0.5
0
0.5
1
1.5
$(dE/dD\text{-}dE/dD|_{D_m})/N^{\theta}$
$N^{\theta}(D\text{-}D_m)$
4-th Step RG
5-th Step RG
6-th Step RG
7-th Step RG
8-th Step RG
9-th Step RG

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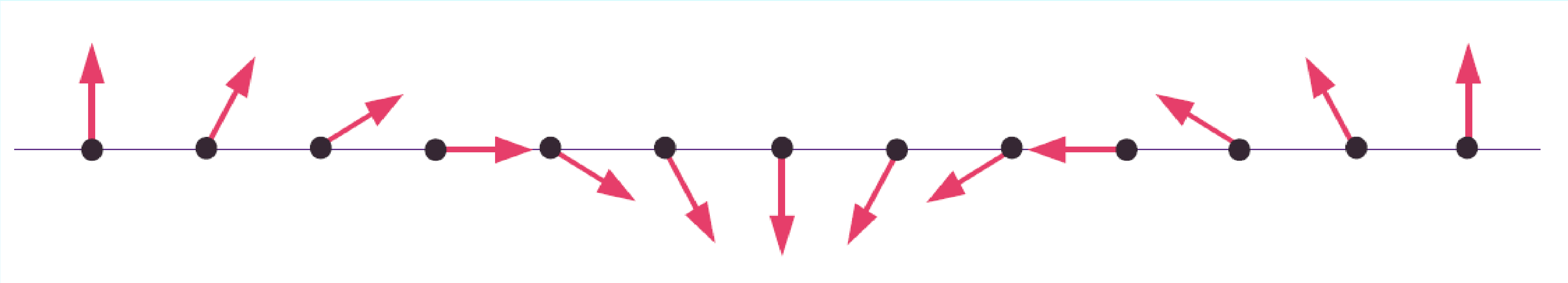
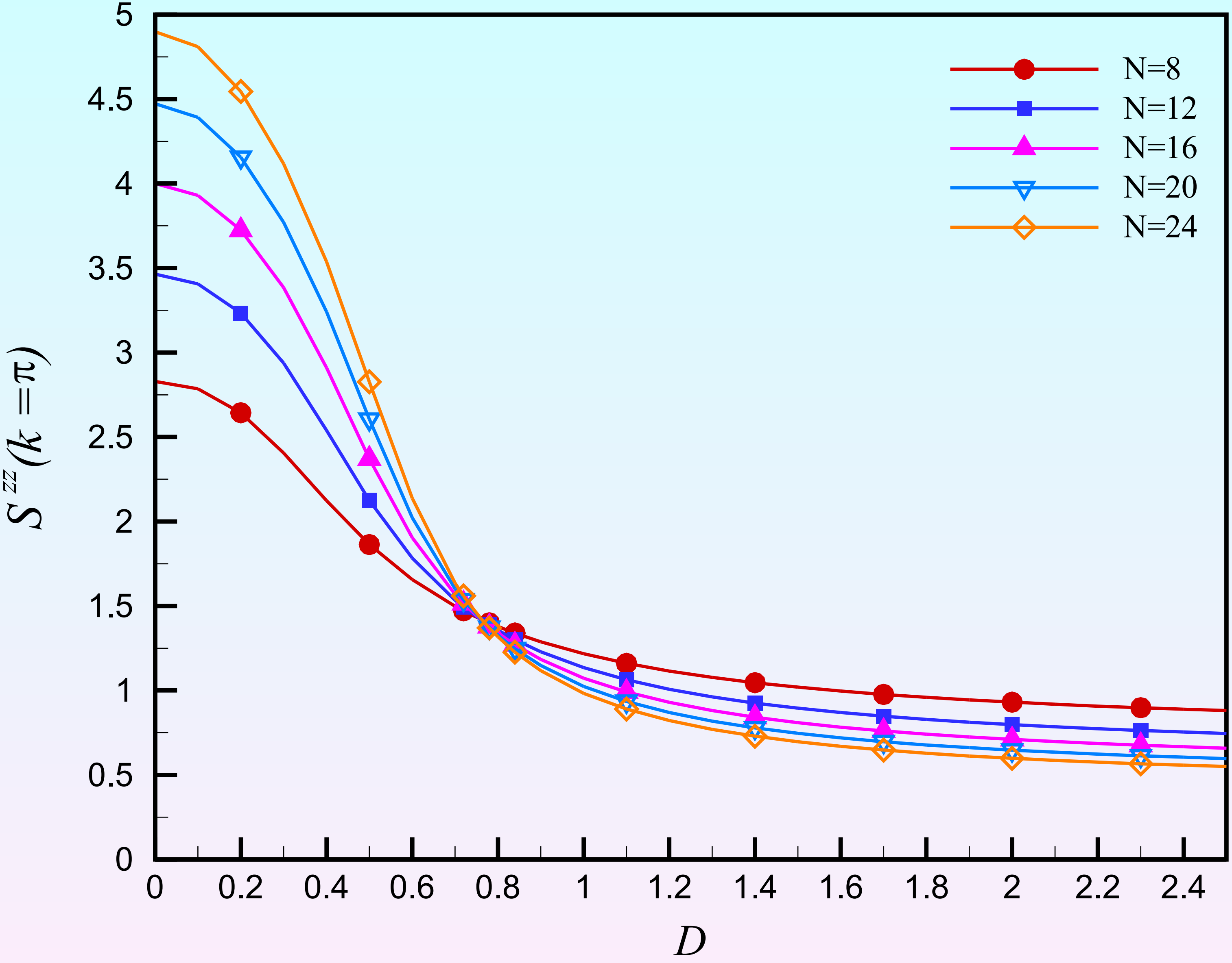
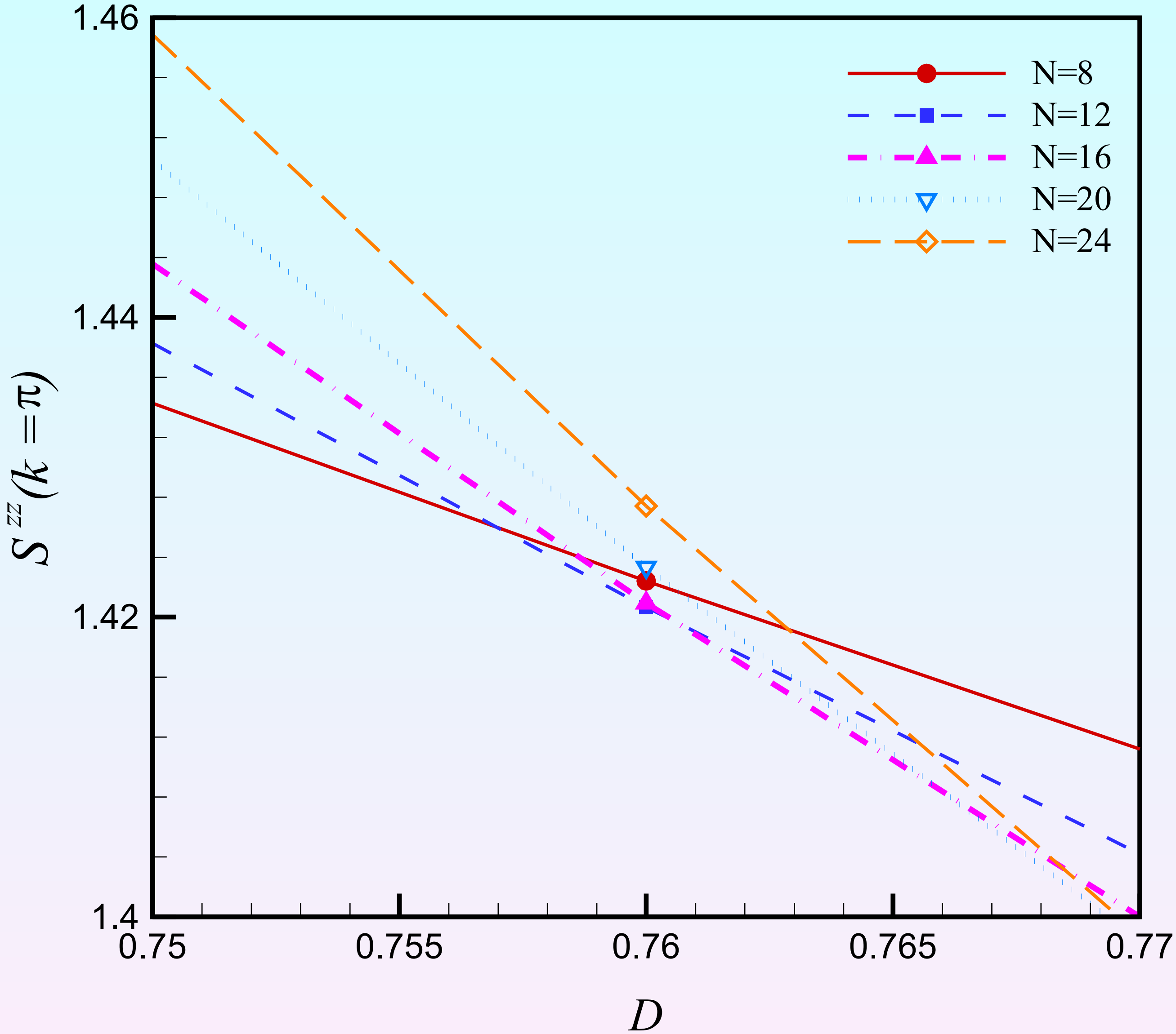
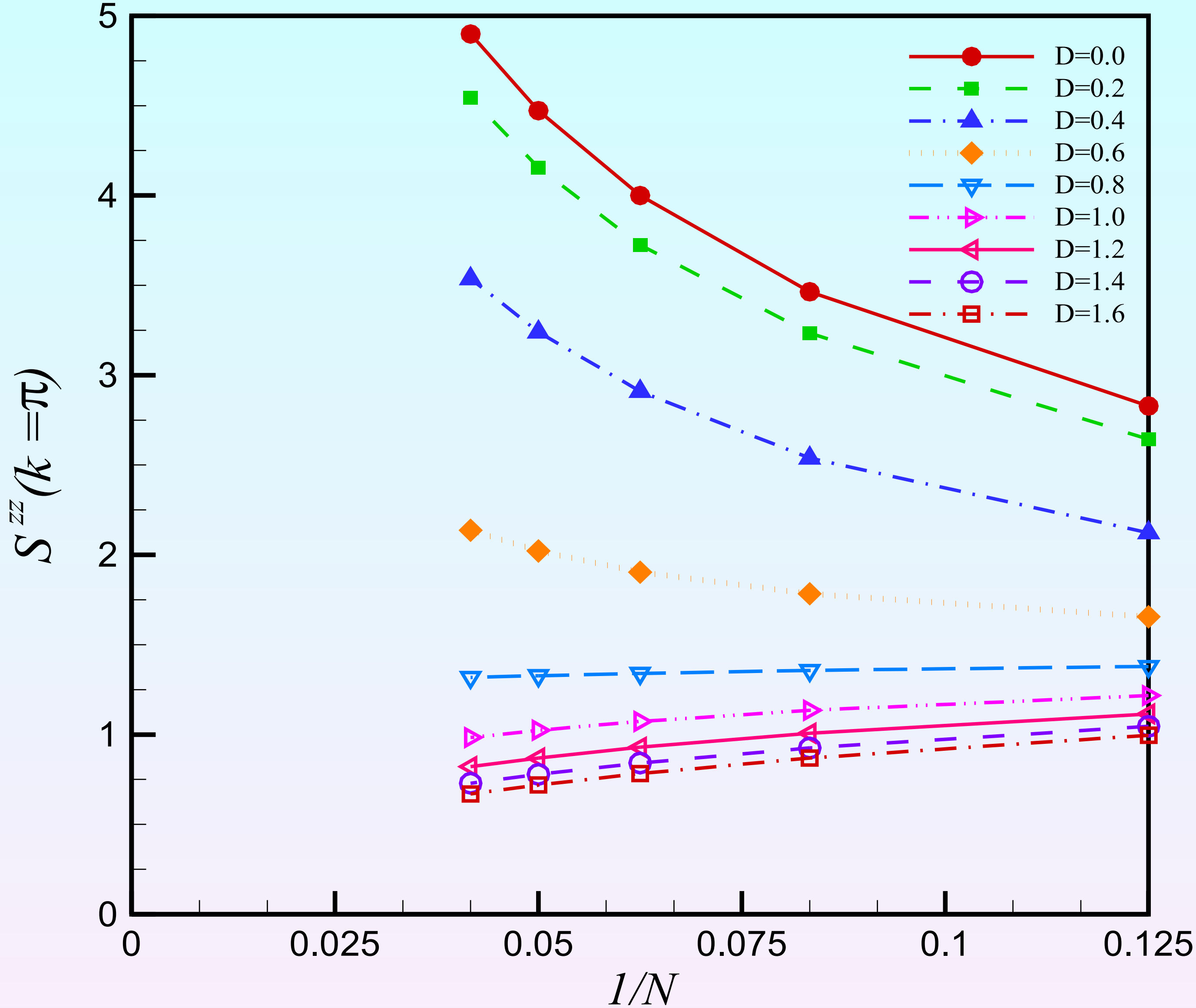
Thank you for your attention



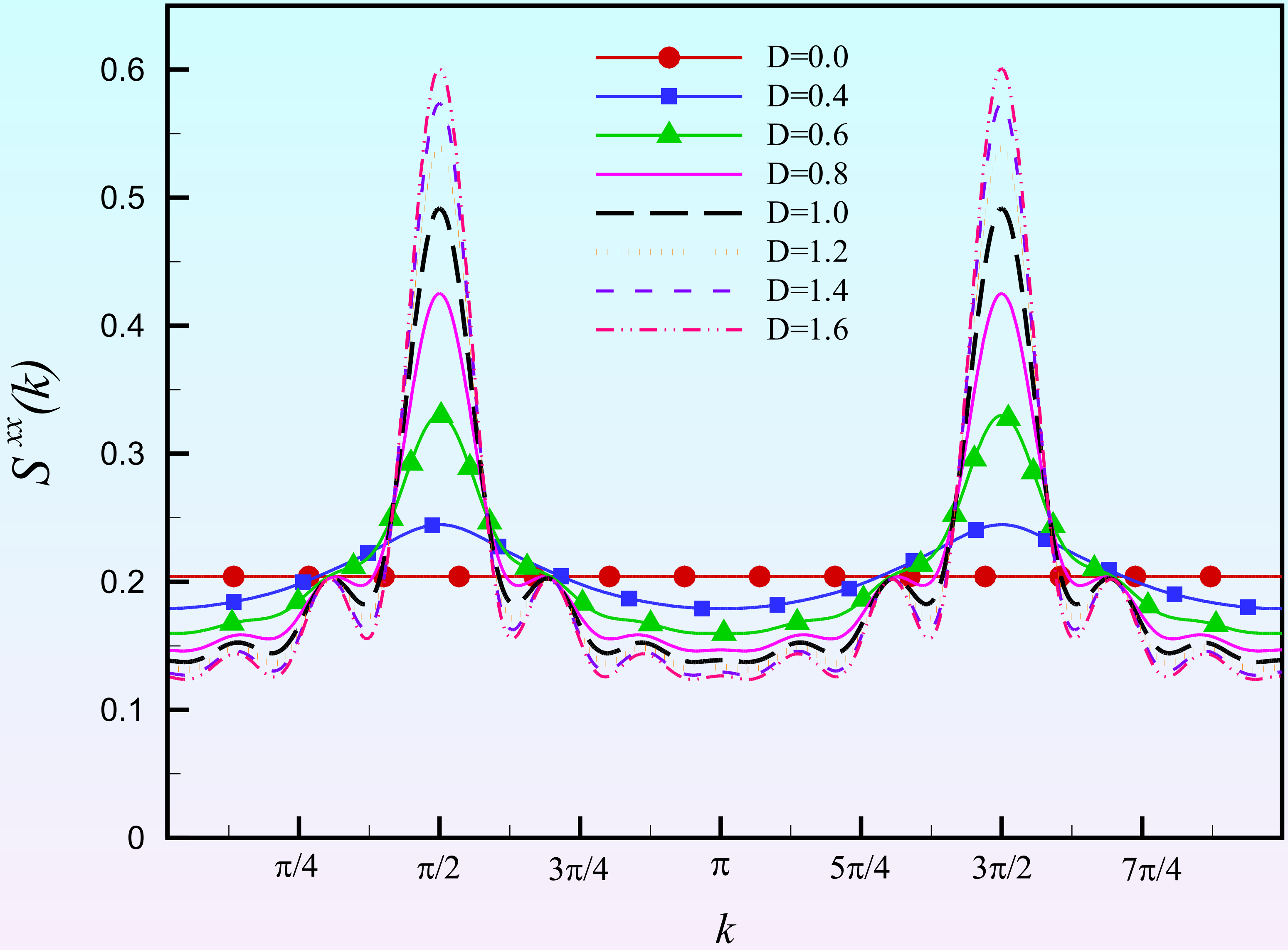


N=8
N=12
N=16
N=20
N=24
$S^{zz}(k=\pi)$
D
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4

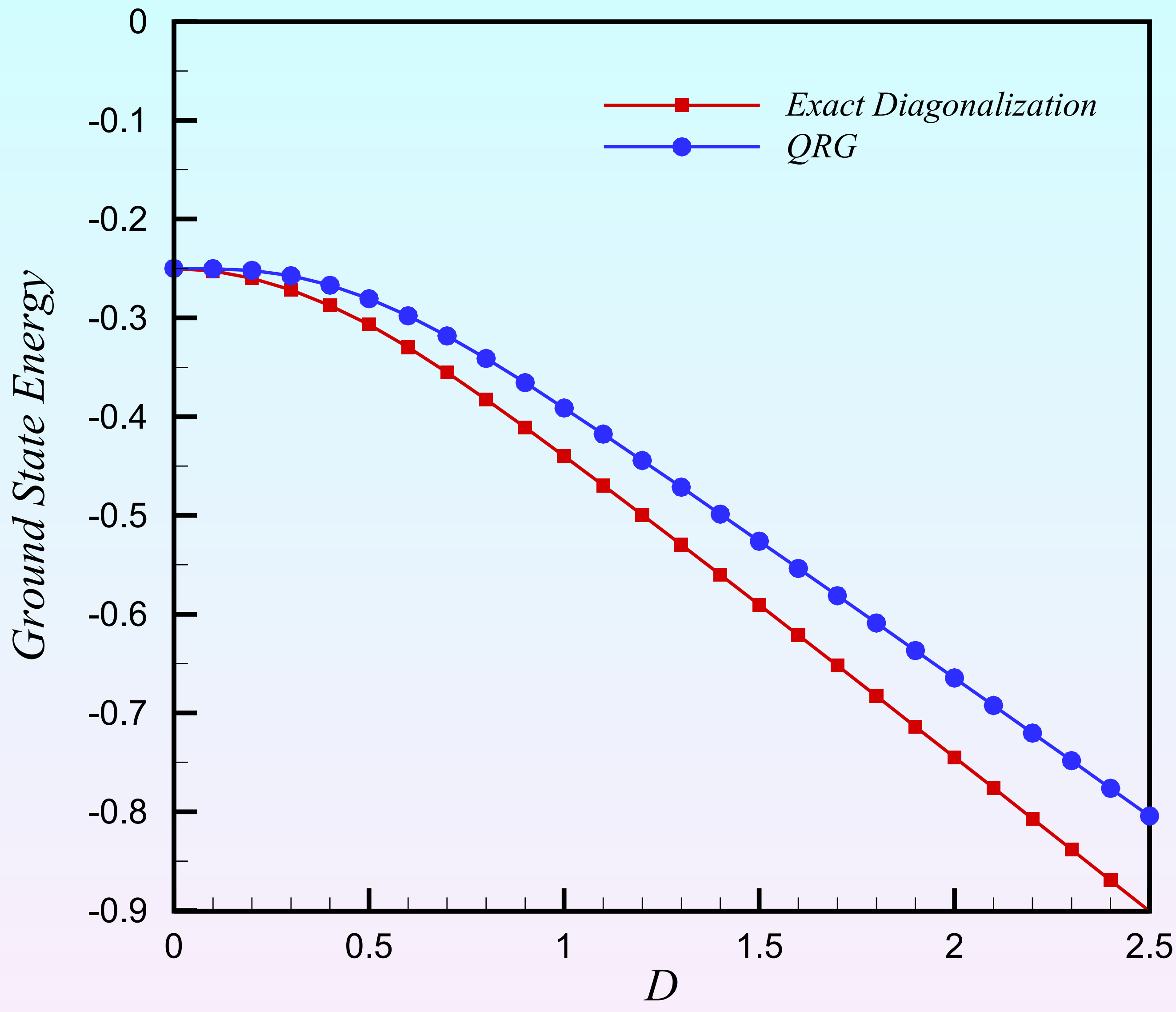

1.46
1.44
1.42
1.4
0.75
0.755
0.76
0.765
0.77
D
$S^{zz}(k=\pi)$
N=8
N=12
N=16
N=20
N=24



D=0.0
D=0.2
D=0.4
D=0.6
D=0.8
D=1.0
D=1.2
D=1.4
D=1.6
$S^{zz}(k=\pi)$
$1/N$
0
0.025
0.05
0.075
0.1
0.125
1
2
3
4
5



D=0.0
D=0.4
D=0.6
D=0.8
D=1.0
D=1.2
D=1.4
D=1.6
0.6
0.5
0.4
0.3
0.2
0.1
0
$S^{xx}(k)$
π/4
π/2
3π/4
π
5π/4
3π/2
7π/4
k


Exact Diagonalization
QRG
Ground State Energy
D
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
0
0.5
1
1.5
2
2.5

$$H^B = \frac{J}{4}\sum_{I=1}^{N/3}\left[\sigma_{1,I}^z\sigma_{2,I}^z + \sigma_{2,I}^z\sigma_{3,I}^z + D(\sigma_{1,I}^x\sigma_{2,I}^y - \sigma_{1,I}^y\sigma_{2,I}^x + \sigma_{2,I}^x\sigma_{3,I}^y - \sigma_{2,I}^y\sigma_{3,I}^x)\right]$$

We have studied the phase diagram and entanglement of the one dimensional Ising model with Dzyaloshinskii-Moriya (DM) interaction. We have applied the quantum renormalization group (QRG) approach to get the stable fixed points, critical point and the running of coupling constants. This model has two phases, antiferromagnetic and saturated chiral phases. We have shown that the staggered magnetization is the order parameter of system and DM interaction produce the chiral order in both phases. Moreover we have analyzed the relevance of the entanglement in the model which let us shed insight on how the critical point is touched as the size of the system becomes large. Nonanalytic behavior of entanglement and finite size scaling have been analyzed which tightly connected to the critical properties of the model.

Phase transition:

- Discontinous (first order) phase transition
- Continous (Second order) phase transition
- Quantum phase transition